Antenna Array Synthesis Using Derivative, Non-Derivative and Random Search Optimization

Stephen J. Blank, *Senior Member, IEEE,* and Michael F. Hutt, *Member, IEEE*

Abstract— **The literature on antenna array optimization has recently focused on direct search methods that use random decision making, e.g., genetic/evolutionary algorithms (GA) and particle swarm optimization (PSO). In this paper we compare results for test problems from the literature using direct search methods that do not use random decision making, such as Nelder-Mead, with those that do, such as PSO. We also revisit gradient based search methods, specifically Newton-like optimization methods that also do not rely upon randomness. Search efficiency is measured in terms of the total number of function evaluations required to converge to an acceptable pattern. For these test problems it is found that direct search without randomness is far more efficient than direct search with randomness; and that non-random gradient based search is most efficient of all.**

Index Terms— **Antenna arrays, optimization, direct search, gradient search.**

I. INTRODUCTION

The literature on antenna array optimization has recently focused on “randomized” direct search methods such as genetic/evolutionary algorithms [1] and particle swarm optimization (PSO) [2]. These methods have become feasible due to the availability of greatly increased computing power. The question arises as to whether or not the assumed greater “exploratory search power” of random decision making search algorithms outweighs their decreased convergence efficiency. Here, search efficiency is measured in terms of the number of function (i.e. array pattern) evaluations required to converge to an acceptable pattern.

Direct methods, depending only on function evaluations but not using randomized search, have long been the method of choice for those problems where it is not feasible to obtain analytic function derivatives. In this paper two direct search method implementations are used: finite difference quasi-Newton search [3] and Nelder-Mead [4] simplex search. The term finite difference signifies the method used to obtain derivative estimates. The Nelder-Mead method, first published in 1965, is one of the best known non-randomized direct search methods.

Derivative (i.e. gradient) based non-randomized search is the classic approach to parameter optimization and is thought to provide increased convergence efficiency as compared to direct (i.e. non-derivative) methods. Gradient based quasi-Newton and Conjugate Gradient search methods have been used extensively for antenna array synthesis.

In this paper we use the test problems specified in [2], and compare the number of function evaluations required to reach the desired results using three different non-random optimization methods.

A. Non-random Search Methods

OPT++ is an open source object oriented nonlinear optimization library [3]. In this paper, two OPT++ routines are used: a finite difference quasi-newton method (FDFNLF1); and a quasi-newton method requiring user supplied derivatives (NLF1).

The Nelder-Mead Simplex algorithm, first published in 1965, is one of the best known non-linear direct search methods. It has proven to be a robust method, and is available in the Matlab optimization toolbox. The C++ version of the Nelder-Mead algorithm, used to obtain the results reported in this paper, was developed by the authors [7].

II. FORMULATION

The problem addressed here is to synthesize a 20 element dual beam reconfigurable array having fixed optimized amplitudes, that provides a pencil beam when the element phases are zero, and a sector beam when optimized non-zero phases are switched on. The pencil beam should have a sidelobe level (SLL) of -30 dB and a beamwidth (BW) at the SLL of 20°. For the sector beam the ripple in the mainbeam, between 78° and 102° should be less than 0.5 dB, and the sidelobes lower than -25 dB between 0° and 70°, and between 110° and 180°. We will solve each problem individually and, then simultaneously.

The antenna array factor for a linear array with the elements aligned along the x axis is given by

$$AF(\phi) = \sum_{n=1}^{N} a_n e^{j\alpha_n} e^{j2\pi cos(\phi) x_n} \tag{1}$$

where a_n is the excitation amplitude, α_n is the excitation phase, and x_n is the x coordinate, normalized to wavelength, of the n^{th} array element. The x_n are fixed and set for half-wave spacing. The set $\mathbf{v} = (a_1, \cdots a_N, \alpha_1, \cdots \alpha_N)$ is the vector of variable parameters used to synthesize a desired pattern. If the array is symmetric, the array factor can be written as

$$AF(\phi) = 2 \sum_{n=1}^{N/2} a_n e^{j\alpha_n} cos(2\pi cos(\phi) x_n) \tag{2}$$

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The array factor in dB is given by

$$P(\phi) = 20\log(AF(\phi)_{normalized}) \tag{3}$$

The mathematical statement of the optimization process is:

$$\text{Find} \min_{\mathbf{v}} f(\mathbf{v}) \rightarrow \mathbf{v_{opt}}$$

where $f(\mathbf{v})$ is the objective function of parameter variables $\mathbf{v}$. For the case of pencil beam synthesis,

$$f_{pencil}(\mathbf{v}) = \sum_{\phi=0^\circ}^{80^\circ} (P(\phi)+30), \text{ if } P(\phi) > Pd1, \text{otherwise } 0 \tag{4}$$

where $Pd1 = -30\text{dB}$ is the desired maximum sidelobe level from 0° to 80°. Here the vector $\mathbf{v}$ consists of variable amplitudes, with all phases set to 0.

For the case of the sector beam synthesis,

$$f_{sector}(\mathbf{v}) = \sum_{\phi=0^\circ}^{70^\circ} (P(\phi)+25), \text{ if } P(\phi) > Pd2$$
$$+ \sum_{\phi=78^\circ}^{90^\circ} (\mid P(\phi) \mid), \text{ if } \mid P(\phi) \mid > Pd3 \tag{5}$$

where $Pd2 = -25\text{dB}$ is the desired maximum sidelobe level from 0° to 70° and $Pd3 = 0.5\text{dB}$ is the maximum desired ripple in the main beam. Here the vector $\mathbf{v}$ consist of both variable amplitudes and phases.

For the case of reconfigurable dual beam synthesis,

$$f_{dual}(\mathbf{v}) = f_{pencil}(\mathbf{v}) + f_{sector}(\mathbf{v}) \tag{6}$$

The vector $\mathbf{v}$ consist of both variable amplitudes and phases and produces both beams. For the pencil beam, the phases are switched off or set to 0, while the optimized phases are used for the sector beam. The amplitudes in this case are the same for both beams.

III. RESULTS

The results of our optimization study are presented in Tables 1, 2 and 3; and Figures 1-6. All three search methods achieve array patterns that satisfy the required performance criteria. The pattern performance results obtained using non-randomized search compare favorable with those reported on in the literature [2] using randomized search. However,in all the cases considered, the non-randomized search methods are significantly better than randomized search in terms of convergence efficiency (i.e. the number of function evaluations). For each of the three synthesis problems considered, the NLF1 analytic derivative based method achieves a solution with the fewest number of function evaluations. The non-randomized direct search methods, FDFNLF1 and Nelder-Mead are a close second. The randomized PSO method is a distant third. As reported in the literature, GA would be fourth. As a measure of the sensitivity of the synthesis results obtained, we plot the excitation amplitudes and phases for the case of dual beam using NLF1 in Figure 6. The amplitude and phase distributions shown in Figure 6 are similar to the results reported in [2].

TABLE I

PENCIL BEAM OPTIMIZATION

	FDFNLF1	NLF1	Nelder-Mead	PSO [2]
Function Evals	212	7	177	5000
Iterations	18	6	115	500

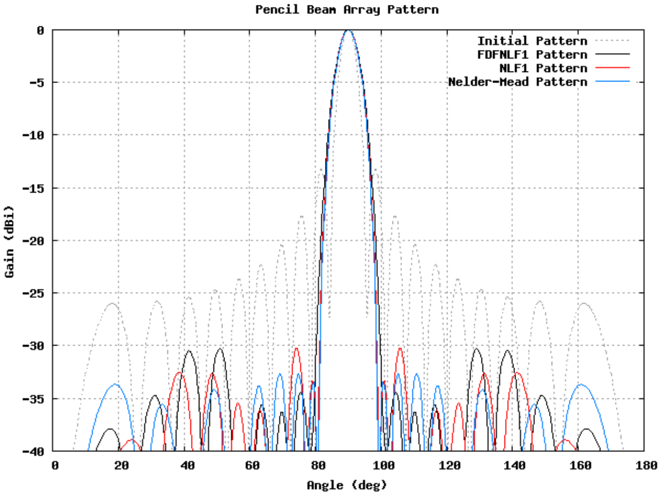


Fig. 1. Pencil Beam Optimization

TABLE II

SECTOR BEAM OPTIMIZATION

	FDFNLF1	NLF1	Nelder-Mead	PSO [2]
Function Evals	879	50	1795	16000
Iterations	41	34	1361	800

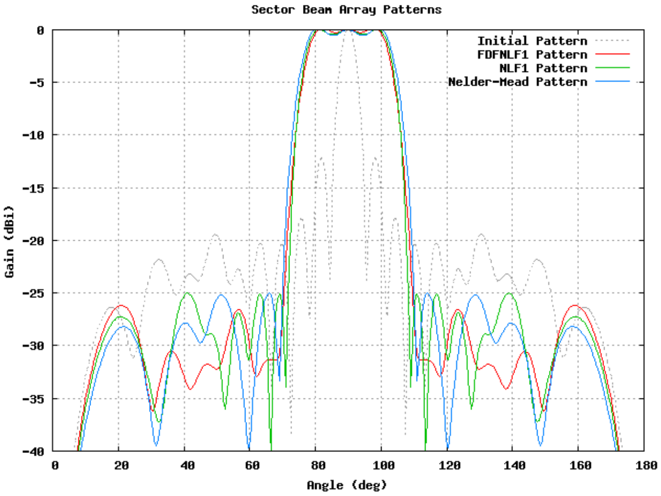


Fig. 2. Sector Beam Optimization

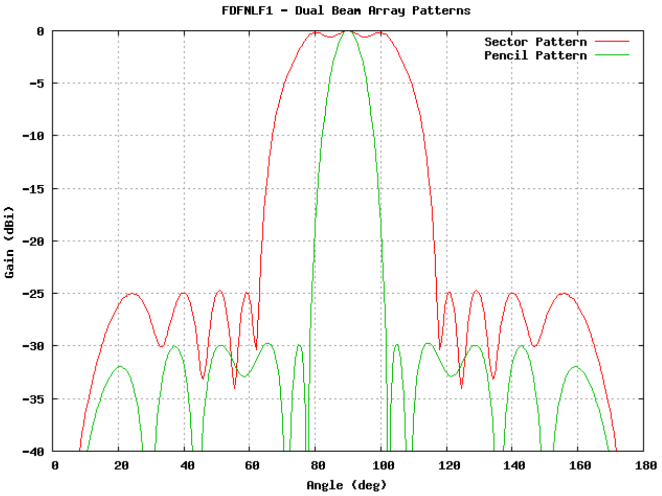


Fig. 3. FDFNLF1 Reconfigurable Dual Beam Optimization

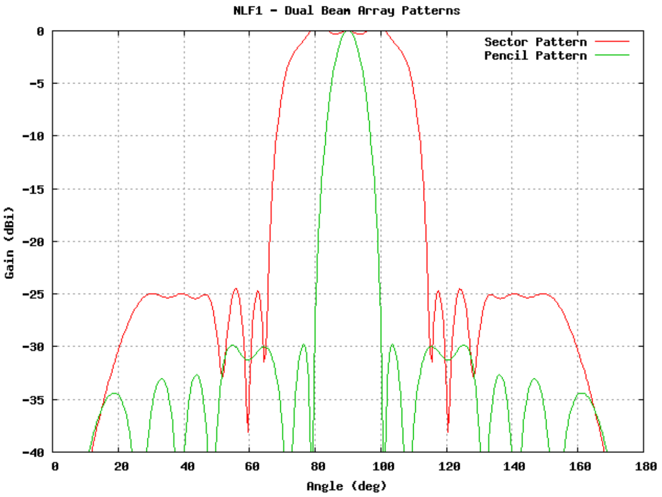


Fig. 4. NLF1 Reconfigurable Dual Beam Optimization

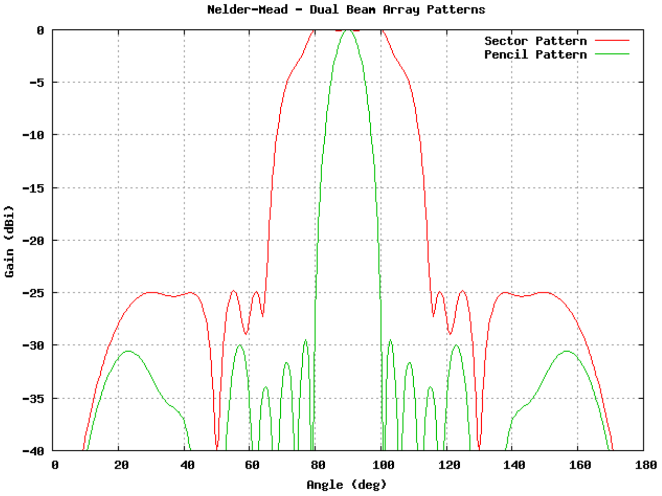
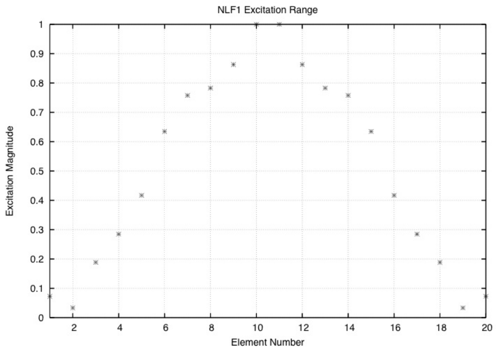


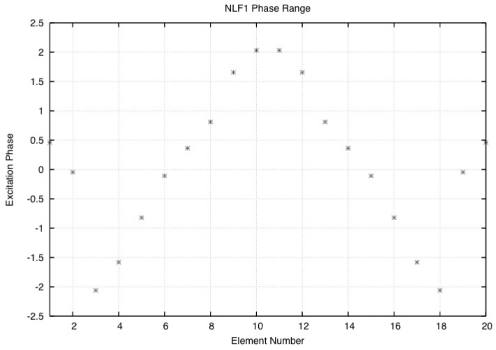
Fig. 5. Nelder-Mead Reconfigurable Dual Beam Optimization

TABLE III
RECONFIGURABLE DUAL BEAM OPTIMIZATION

	FDFNLF1	NLF1	Nelder-Mead	PSO [2]
Function Evals	2147	123	2542	21000
Iterations	100	100	1960	1100



(a) Excitation Amplitudes



(b) Excitation Phases

Fig. 6. NLF1 Optimum Amplitude and Phase Distributions

IV. CONCLUSION

For the three antenna array synthesis problems considered in this paper, non-randomized search methods required far fewer function evaluations to converge to acceptable pattern performance than did the randomized search method. It might be said that this result was to be expected. It's interesting to note that in order to overcome the convergence difficulties of random based optimization methods, Nelder-Mead simplex has recently been combined with PSO and GA, into hybrid optimization algorithms [5] [6]. The results reported here indicate that for these types of synthesis problems, gradient and non-gradient methods without randomness seem to be significantly more efficient in achieving a desired pattern.

A method for antenna array synthesis under realistic conditions accounting for mutual coupling effects is discussed in [8]. The approach taken by [2] of incorporating closed form synthesis procedures, such as the Woodward-Lawson method, into an optimization search algorithm could be a subject for future research.

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Stephen J. Blank Stephen Jon Blank, Ph.D., has worked on research and development in the field of antennas, microwaves and electromagnetics for many years. At AIL, L.I., NY, he worked on circularly polarized horns, surface wave antennas, broadband baluns and ELINT techniques. At the Grumman Aerospace Corp, L.I., NY, he was Antenna Group Leader on the F-111B project and co-inventor of the circularly polarized trihedral reflector, used for automated carrier landings by the U.S. Navy, U. S. patent 3,309,705. In 1984, he received a NASA Certificate of Recognition for developing the algorithm for the Empirical Optimization of Antenna Arrays. The computer code that he developed to implement the empirical optimization algorithm was part of NASAs computer library, COSMOS. At JPL, Pasadena, CA, he worked on the design of array feeds for distortion correction and vernier beamsteering of large reflector antennas. In 1982 he founded a consulting company, IBE Systems; working mainly in the field of broadcast engineering, SW/AM/FM radio and TV, in the U.S. and overseas. He co-authored the chapter on Directional Couplers for the Handbook of Microwave Technology, Academic Press, 1995. In 1996, he worked on the development of array antennas for MRI systems. In the period 2002- 2003 he conducted a study of the use of microwave beaming to supply power to an unmanned aerial vehicle for NASA.

He has had various papers published in the IEEE Transactions on Antennas and Propagation and has also served as a reviewer. From 1989 to 1994 he was chairman of the IEEE Antennas and Propagation Society, NY/LI section. In 1992 he was technical chair and in 1994, chairman-at-large, of the IEEE Symposium on Adaptive Arrays. He was technical chairman of the Task Force on Energy Conservation and Radiation Hazard Reduction, IEEE LI section. In 1995, he received the IEEE Region 1 Electrical Engineering Professionalism Award for his contributions to energy conservation and radiation hazard reduction. In 1998, he was invited to lecture on the subject of Vertical Polarization for FM Broadcasting at the 21st Congress of the Association of Brazilian Broadcasters in Iguacu, Brazil. He is a Professor of Electrical and Computer Engineering at the New York Institute of Technology, Old Westbury, NY.

Michael F. Hutt Michael F. Hutt holds a BS and MS in Electrical Engineering and is President of Hutt Systems, Inc. He is currently developing software applicable to the optimization of linear, planar, and conformal antenna arrays, which includes the implementation of direct search and gradient based (optimization) methods. He also has experience with mixed-language programming and has investigated optimization methods as applied to impedance matching networks. In 2003 he developed the Multicast Management Tool for Router Management Solutions, Inc. which he co-founded. This application has since become Cisco Multicast Manager. Prior to this he developed custom network management solutions for Financial companies in the New York City area. In addition to developing these web based applications, he also configured and administered the web servers on which they ran. He has taught classes in High Speed Information Networks and C++ at New York Institute of Technology, and has provided networking and Perl training classes for businesses. He is a member of the ACM and IEEE.