Transmitter Illumination Taper as a Design Parameter for Wireless Power Transmission Systems

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***Abstract*—Wireless power transmission (WPT) is being studied for a variety of important applications. This paper presents a technique to compute the effect of varying transmitter aperture illumination tapers on beaming efficiency and on the power density variation incident on the receive aperture of a WPT system. Results are presented showing the effects on beaming efficiency and on incident power density variation. The optimal choice for transmit illumination taper is discussed.**

***Index Terms*—Wireless Power Transmission, aperture illumination, power density, millimeter waves.**

I. INTRODUCTION

Wireless power transmission (WPT) is being studied for a variety of important applications [1,2,3]. Prominent among these applications is the satellite solar power project to beam gigawatts of electric power from solar panels in geosynchronous orbit down to the earth [4,5]. On a smaller scale, there are studies of the use of WPT to power large, unmanned airships stationed in the stratosphere at a range of 20 km [6]. Such fixed station platforms can serve as telecommunication centers as an alternative to satellite communication systems. On an even smaller scale are the studies of the use of WPT to power relatively small, unmanned aerial vehicles (UAV) over ranges in the order of 0.5 to 5 km [7]. Such low altitude UAVs can serve a variety of civil and military functions. The frequencies being considered for WPT operation, range from 2.45 MHz and 5.8 MHz in the microwave region, through millimeter wave frequencies up to infrared. In the millimeter wave range, 35 and 94 GHz are at relatively lower atmospheric attenuation windows. If portability of the WPT system is important then 94 GHz is advantageous since the size of the transmit and receive antenna apertures decrease with increasing frequency.

WPT systems transfer electrical power from one location to another without the use of cables. A WPT systems can do this for indefinitely long periods of time and without the need for, usually heavy, storage units at the receive site. The first step in the WPT process is to convert energy from a DC (or AC) source to RF energy in a transmitter. The second step is to beam this energy wirelessly through the atmosphere over some specified distance to a receive aperture. The efficiency in doing this is called the beaming efficiency. The third step is to collect this energy and to rectify the RF energy to DC output energy. The subsystem for this third step consists of an array of antenna and diode rectifier units and is called a rectenna. The efficiency in doing this is called the rectenna efficiency. At infrared, photovoltaic cells may replace diode rectifiers.

In this paper we investigate the influence of transmitter aperture illumination on the overall efficiency of wireless power transmission (WPT) systems. Assuming a Gaussian transmit aperture field distribution, we study the effects of varying transmit illumination tapers on i) beaming efficiency and on ii) power density variation incident on the receive aperture. Although the technique presented can be applied to a variety of WPT systems, the specific motivation for this study of optimal beaming efficiency is to provide wireless power to an unmanned aerial vehicle. In addition to finding the best taper to optimize beaming efficiency, we are also concerned with minimizing the variation in incident power density at the receiver. Excessive variation in the incident power density at the receiver will result in sub-optimal performance of the rectenna elements. We present results showing the effect of various transmit aperture tapers on beaming efficiency and incident power density taper.

II. FORMULATION

In this discussion we assume that it is desired to wirelessly beam power from a circular transmitter aperture of diameter D_t to a circular receive aperture of diameter D_r - see Fig. 1. The relationship between D_t, D_r, the range between them, r_o, and the wavelength at the frequency of operation λ, is given by the following simple rule of thumb [8].

$$D_t = 2.44\frac{\lambda r_o}{D_r} \tag{1}$$

From (1), for the case of $D_r = 1.4m$, and $r_o = 1km$, we see that the transmit aperture diameter becomes prohibitively large at frequencies below 10 GHz. For applications that require a mobile transmitter, 94 GHz would be a practical choice.

Electromagnetic energy traveling through the atmosphere suffers from atmospheric attenuation caused primarily by absorption by atmospheric gasses. For lower frequencies (below 10 GHz), the attenuation is small to insignificant and reasonably predictable [9]. For high frequencies in the millimeter wave range, the attenuation not only increases, but becomes more dependent upon peculiar absorbing characteristics of H_2O, O_2, etc. The intensity of precipitation can strongly affect atmospheric attenuation. High levels of precipitation can cause severe attenuation. However, a statistical analysis of meteorological records show that the probability of occurrence of such levels of precipitation is sufficiently low, so as to not pose a serious problem for the proposed application [8].

Referring to Fig. 1, we assume that the electric field in the circularly symmetric transmit aperture has a Gaussian distribution given by:

$$Ea(r') = e^{-\left[\frac{r'}{w(T_t)}^2\right]} \tag{2}$$

where

$$w(T_t) = \frac{0.5D_t}{\sqrt{-ln\left(10^{\frac{T_t}{20}}\right)}} \tag{3}$$

and where T_t is the transmit aperture edge taper in dB, i.e., the value in dB of the aperture illumination at the edge divided by the aperture illumination at the center,

$$T_t = 20log\left(\frac{Ea(a)}{Ea(0)}\right)$$

As previously reported [10], Gaussian illumination is near optimum for the cases being considered.

In order to emphasize the dependence of our result on the parameter T_t we designate the aperture illumination by $Ea(r', T_t)$. In what follows, computational results will be presented showing the effect of various values of transmit illumination taper, T_t, on beaming efficiency and on power density variation incident on the receive aperture.

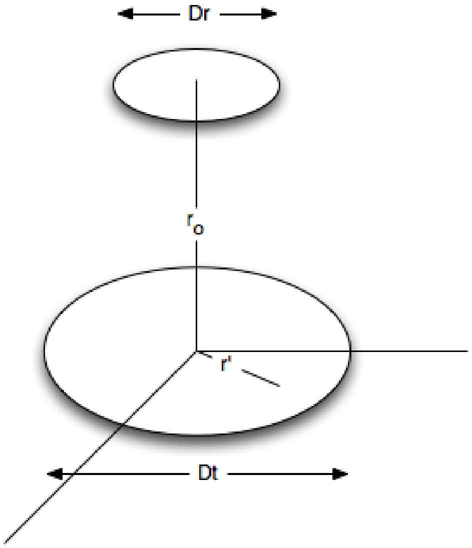

Fig. 1. Power Beaming Geometry

A. The Radiated Field

The electric field radiated by a circular aperture is given by the diffraction integral as follows, see Fig.2:

$$E(\theta, T_t) = \int_0^{2\pi}\int_0^{a} E_a(r', T_t)e^{-j(kr''-\alpha)}r'\mathrm{d}r'\mathrm{d}\phi'\frac{-j}{\lambda r_o} \tag{4}$$

where

$$r'' = \sqrt{r_o^2 + r'^2 - 2r_o r' sin(\theta)cos(\phi')}$$

and $a = 0.5D_t$. The term α represents the aperture illumination phasing required to focus the radiated field at the finite range r_o.

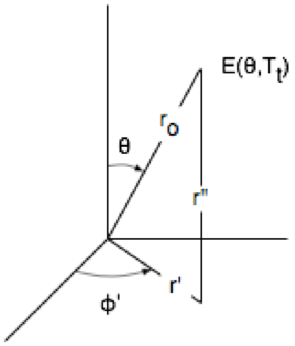

Fig. 2. Coordinate System

The total power radiated, $P_{rad}(T_t)$ is given by:

$$P_{rad}(T_t) = \frac{\pi}{377}\int_0^{a} E_a(r', T_t)^2 r'\mathrm{d}r' \tag{5}$$

The total power incident on the receive aperture, $P_{inc}(T_t)$, is given by:

$$P_{inc}(T_t) = \frac{\pi}{377}\int_0^{\theta_0} |E(\theta, T_t)|^2 r_o^2 sin(\theta)\mathrm{d}\theta \tag{6}$$

In the above, computations are simplified by taking advantage of the circular symmetry of the beaming configuration.

III. RESULTS

The beaming efficiency, $\eta_b(T_t)$, as a function of transmit aperture illumination taper, T_t, is defined as:

$$\eta_b(T_t) = \frac{P_{inc}(T_t)}{Prad(T_t)} \tag{7}$$

A graph of $\eta_b(T_t)$ for T_t varying for -0.5 to -24 dB is shown in Fig. 3.

Another important beaming factor is incident power density taper, T_r, at the receive aperture. T_r as a function of T_t is defined by:

$$T_r = 20\log\left(\frac{|E(\theta_0, T_t)|}{|E(0, T_t)|}\right) \tag{8}$$

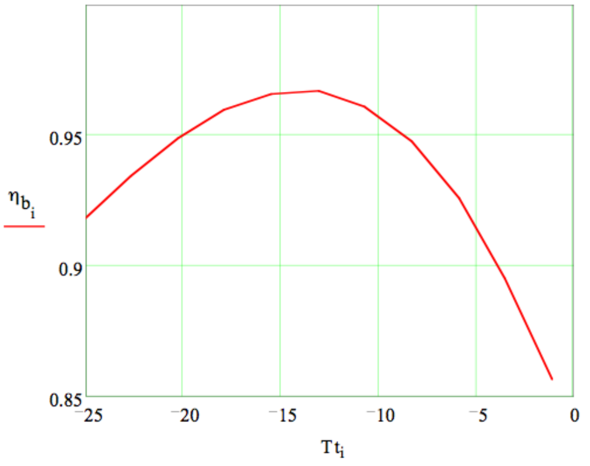

Fig. 3. η_b as a function of Tt in dB

The incident power density taper, T_r, is important because the receive aperture contains the wireless power systems' rectenna which comprises an array of small receive antennas each terminated in a diode. The rf to dc conversion efficiency of a diode is dependent on the level of the incident power density. The diode rf to dc conversion efficiency is very low at low levels of incident power density.

A graph of T_r as a function of T_t is shown in Fig. 4.

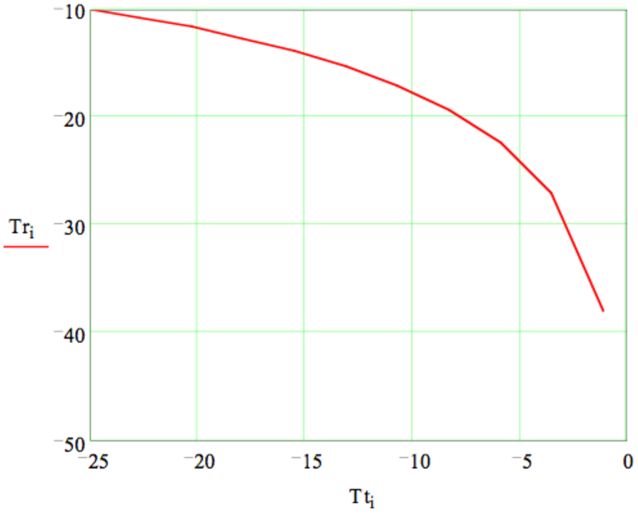

Fig. 4. Tr in dB as a function of Tt in dB

IV. CONCLUSION

The overall total efficiency of a wireless power beaming system depends on the efficiency of each of its subsystems. In this paper we have reported on the effect of transmit aperture illumination taper T_t on beaming efficiency, η_b, and incident power density T_r at the receiver.

Computational results are presented graphically in Fig. 4 and Fig.5. The numerical results show that the maximum beaming efficiency value of $\eta_b = 0.97$ is obtained for a value of $T_t = -14dB$. This result must be balanced against the results computed for the incident power density taper, T_r, as a function of T_t. The computations show that $T_r = -15dB$ when $T_t = -14dB$. Higher values of T_t result in lower values of T_r. Therefore, the system designer must choose an optimum compromise between high beaming efficiency, η_b, and low incident power density taper, T_r. The designer must account for the effect of incident power density variation at the receive aperture on optimum diode rectification efficiency. For example, such an analysis might lead to specifying variations in the sizes of the receive antenna apertures of the individual rectenna elements as a function of their position in the receive rectenna array. In addition to the overall system efficiency, environmental constraints must also be satisfied. It can be shown using the method described above that the power density at the edge of the receive aperture is an upper bound on incident power density in the near vicinity of the rectenna aperture. These questions concerning beaming characteristics along with optimum rectifier design will be dealt with in subsequent phases of this research.

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